

ORCA Math Conference for
Teachers & Undergraduates
Titles and Abstracts

Invited Speaker

Speaker: Lisa Berger, Stony Brook University

Title: Shelving Books

Abstract: Your new apartment has built-in bookshelves, and you want to know whether or not your mathematical tomes will fit. Is it enough to know the length and number of the shelves and the total thickness of your books? More formally, for a positive integer L , $m > 1$, positive integers v_i , and a solution $\{a_i\}$ to the non-negative Diophantine equation: $mL = \sum_{i=1}^n x_i v_i$, when can we find b_i , with $0 \leq b_i \leq a_i$, satisfying $L = \sum_{i=1}^n b_i v_i$? We'll discuss additional applications of this problem, partial results, and open questions. This shelf-packing problem, a special example of the partition problem, would serve as a nice exploration for high school students and their teachers.

Contributed Talks

Title: The 1-Relaxed Game Chromatic Number of Paths and Extended Stars

Speaker: Lynnette Snyder (Linfield College, Willamette Valley REU-RET)

Abstract: We consider the $(2, 1)$ -relaxed coloring game on graphs. Two players, Alice and Bob, color a graph G with 2 colors. Alice has the first move. A color is legal for a vertex, x , if that vertex has at most 1 neighbor, w , the same color, and w had no neighbors colored the same prior to x being colored. Alice wins if every vertex is colored, while Bob wins if at some point a vertex has no legal color. We show that Alice has a winning strategy for this game on paths, extended stars, and more general cases with extended stars.

Title: One Relaxed Game Chromatic Number of Caterpillars

Speaker: Mallory Schaffert (Linfield College, Willamette Valley REU-RET)

Abstract: We analyze the $(2, 1)$ -relaxed coloring game. Two players, Alice and Bob, are given two colors to color a graph. They take turns coloring the uncolored vertices with legal colors. A color α is legal for an uncolored vertex u if u is adjacent to at most 1 vertex v that has already been colored with α , and v is not adjacent to any vertices already colored α . Alice wins the game if eventually all of the vertices are legally colored; otherwise Bob wins the game when there is an uncolored vertex with no legal color. We show that Alice has a winning strategy for a specific class of caterpillars when playing this game.

Title: A Tree with Maximum Degree 3 and Game Chromatic Number 4

Speaker: Victor Larsen (Linfield College, Willamette Valley REU-RET)

Abstract: We examine the following coloring game played on a graph. Given a fixed positive integer r , Alice and Bob alternately color the vertices of G with r colors. Alice makes the first move. The game ends when no more vertices can be legally colored. A color α is legal for a vertex u if no neighbors of u are colored with α . If the entire graph is eventually colored, Alice wins. If there exists any uncolored vertex with no legal color, then Bob wins. The least r such that Alice has a winning strategy is called the game chromatic number of G . This parameter has been examined in many other papers, and it has been proven that every tree has game chromatic number at most 4. We show that there exists a tree with maximum degree 3 on which Alice does not have a winning strategy using only 3 colors. This proves that there exists a tree with maximum degree 3 with game chromatic number 4.

Title: Clique-Relaxed Graph Coloring

Speaker: William Sehorn (Linfield College, Willamette Valley REU-RET)

Abstract: We define a variation of the chromatic number of a graph G called the k -clique-relaxed chromatic number, denoted $\chi^{(k)}(G)$. We prove that $\chi^{(k)}(G) \leq \left\lceil \frac{\chi(G)}{k} \right\rceil$ for all graphs G , and we prove corollaries for outerplanar and planar graphs. We also define the k -clique-relaxed game chromatic number, $\chi_g^{(k)}(G)$, of a graph G . We prove $\chi_g^{(2)}(G) \leq 4$ for all outerplanar graphs G and give an example of an outerplanar graph H where $\chi_g^{(2)}(H) \geq 3$.

Title: Obstacle Number of Graphs

Speaker: Christina Koch (Willamette University, Willamette Valley REU-RET)

Abstract: An obstacle representation of a graph G is a drawing of G in the plane together with a set of polygons called obstacles, where an edge exists in G if and only if it does not intersect an obstacle. The smallest number of obstacles required to represent G in the plane is the obstacle number of G . Previous research about obstacle number seemed to suggest that most, if not all graphs had obstacle number one. Among the results presented in this talk will be a construction that guarantees arbitrarily large obstacle number, contradicting this early conjecture. We will also present various classifications of graphs and their obstacle number.

Title: Convex Obstacle Numbers of Graphs

Speaker: Hannah Alpert (Willamette University, Willamette Valley REU-RET)

Abstract: What if we consider only those obstacle representations with all obstacles convex? The convex obstacle number of a graph is the smallest number of obstacles in any such representation. Convex obstacle number can be quite different from obstacle number: for every k , there is a graph with obstacle number 1 and convex obstacle number k . Also, surprisingly, the classes of graphs with convex obstacle number 1 and those representable with a single line-segment obstacle are both equivalent to previously-studied classes of intersection graphs.

Title: Expected Length of Minimal Spanning Trees on Graphs With Randomly Weighted Edges

Speakers: Krista Foltz, Chelsea McLennan, Jared Nishikawa, Ryan Valles, and Nada Bushey (Willamette University, Willamette Valley REU-RET)

Abstract: The driving force behind our project is the following: given a graph with random edge weights, can we effectively make quantitative statements about its minimal spanning tree? What effects do edge distributions or the structure of the graph have on the expected length of the minimal spanning tree?

Steele's Formula is our primary method for finding the expected length of the minimal spanning tree of a graph via use of the Tutte polynomial, $T(G; x, y)$, of a graph. Given a graph G and edge weight distribution $F(t)$, the expected length of the minimal spanning tree, $E[L_{MST}^F(G)]$, is given by Steele's formula:

$$E[L_{MST}^F(G)] = \int_0^\infty \frac{1 - F(t)}{F(t)} \frac{T_x(G; x, y)}{T(G; x, y)} dt.$$

The Tutte polynomial can be derived using a deletion-contraction algorithm.

So far, we have had success in deriving the expected length of the minimal spanning tree for particular families of graphs (cycle graphs, flower graphs, and their variations, etc..) and have obtained evidence to support conjectures for general graphs.

Title: Classifying Graphs as Intrinsically Knotted

Speaker: Chris Morris (CSU, Chico)

Abstract: With the help of Dr. Mattman, I have been working on classifying seven, eight and nine vertex graphs with respect to the property of intrinsic knotting. We have taken a brute force, programmatic approach to our classification efforts. By encoding all known techniques for classifying graphs with respect to this property, we have created a suite of tests to perform on every graph. This talk will discuss this process and progress to date, as well as describe knots, graphs and the property of intrinsic knotting.

Title: Intrinsic Knotting of Bipartite Graphs
Speaker: Sophy Huck (CSU, Chico REUT)

Abstract: We identify and categorize intrinsically knotted bipartite graphs. We are motivated by a conjecture that a bipartite graph with $E \geq 4V - 17$ is intrinsically knotted. We verify the conjecture for graphs that have exactly 6 vertices in one part and at least 6 in the other. We also provide similar bounds for all bipartite graphs.

Title: Focusing UCT Search in Go

Speakers: Jason Galbraith, Andrew Hubbard, Seth Pellegrino, John Stogin, and Kevin Imber (Lewis & Clark College, Willamette Valley REU-RET)

Abstract: Writing programs to play the Asian game of Go is considered a grand challenge in the field of artificial intelligence. There has been significant progress in recent years due to the UCT algorithm, which uses Monte-Carlo sampling and biases future samples based on the results of past samples. Our work explores modifications to UCT which further focus computational effort on critical parts of the game.

Title: From Simple Continued Fractions to λ -Fractions

Speakers: Shira Hetz and Matthew Bracher (CSU, Chico REUT)

Abstract: Continued fractions provide a useful way to approximate real numbers by nested fractions with positive integer denominators. Following the work of Rosen et al., we examine the properties of generalized continued fraction expressions where the integer partial quotients are replaced by integer multiples of an algebraic number, λ . Our group uses Rosen's construction to characterize units in certain real quadratic extensions of the rational numbers.

Title: Ergodic Methods in the Theory of λ -Fractions

Speaker: Michael Ontiveros (CSU, Chico REUT)

Abstract: Ergodic theory is used to describe the properties of a measure preserving transformation on a probability space. Recent years have shown it especially useful in number theory, and for the purposes of our research, in the theory of continued fractions. For this talk we define a coefficient-generating transformation for λ -fractions and, following the work of R. Burton, C. Kraaikamp, and T. Schmidt, show how we might use ergodic methods to describe the structure of λ -fraction representations for almost all real numbers.

Title: Arithmetic Derivatives of Integers

Speaker: Aaron Sewell (CSU, Chico REUT)

Abstract: This study is involved in a relatively new field of study involving the arithmetic derivative of integers. Our work is based in a 2003 paper of Ufnarovski and Ahlander paper describing the basic properties and theorems published. In this talk we will concentrate on the few studied properties of the anti-derivatives of the integers. In particular, we will define special families of " n -th order" anti-derivatives of Integers and addresses issues of integers without anti-derivatives.

Title: Mathematical Modeling of Rotational Flows of Visco-Elastic Fluids

Speakers: Christine Bittner, Konstantin Dragomiretskiy, Jim Friedrich, and Kris Kilpatrick (CSU, Chico REUT)

Abstract: Rimming flow of the non-Newtonian visco-elastic fluid on the inner surface of a horizontal rotating cylinder is investigated. After the scale analysis of the complete set of equations, which model the basic laws of mass and momentum conservation, the simplified model of the thin film flow was derived. Applying the method of perturbations the mathematical model reduces to simple algebraic equations regarding the liquid film thickness. The qualitative and quantitative analysis of this equation is carried out and the approximate solutions are obtained. Based on these solutions, different regimes of the rimming flow are defined and analyzed analytically and numerically.

Title: Identification of Individual Great Horned Owls Using Wavelet Analysis

Speakers: Robert Vandermeulen, Katherine McCaffrey, Kevin Schiedler, Nicole Kingsley, and Heather Thomas (University of Portland, Willamette Valley REU-RET)

Abstract: In an attempt to track individual Great Horned Owls without the obstacles involved in tagging, voice recognition may be an efficient alternative. By recording owl hoots and studying their waveforms, mathematical methods can be used to determine unique characteristics. Traditionally, analysis of these signals has employed Fourier techniques which localize the signal in the frequency domain. In recent decades, however, Wavelet Analysis has become an effective method of examining a signal in both the frequency and time domains. We use wavelet packet decomposition to identify characteristics in hoots that are unique to each individual owl.

Title: Determining the Social Class Backgrounds of UC and CSU Mathematics Faculty

Speakers: Sophy Huck, Carol Wilson, Megan O'Connor, and Elizabeth Zapata (CSU, Chico REUT)

Abstract: We present a definition of “social class” that is based on a review of the research literature in mathematics education. Using this definition, we examine the social class backgrounds of all tenured/tenure-track mathematics professors in the UC and CSU systems. Current research confirms that there is a shortage of minority and female professors in mathematics, and that this fact has implications for female and minority undergraduate students’ access to role models who are from backgrounds similar to their own. This pilot study aims to determine the availability of role models to UC and CSU mathematics students from low social class backgrounds.