

Learning Outcomes for Mathematical Reasoning for the Baccalaureate Degree

Learning Outcomes Project Final Report
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II. Preface

The objective of this project is to “formulate a model pilot set of basic learning outcomes for mathematical reasoning and written communication in English” for the California State University (CSU) system (Ref. 1). The outcomes developed will be expectations for minimum exit standards applicable to all students who graduate from a CSU campus with a baccalaureate degree, regardless of course of study. The defined outcomes should preferably be assessable. This document addresses learning outcomes in mathematical reasoning.

There are four general aspects to producing a mathematically literate CSU graduate. For purposes of this study, these are defined using the following vernacular:

- a comprehensive learning *objective*;
- specific learning *outcomes*, which in aggregate are intended to comprise the objective;
- educational *goals*, enacted to produce the desired outcomes; and
- an *assessment* procedure, to evaluate the successes and deficiencies within the entire process and to provide guidance in making improvements.

The driving force behind the development of outcomes for the CSU is *The Cornerstone Report* (Ref. 2), wherein the CSU is called upon to “award the baccalaureate on the basis of demonstrated learning, as determined by [its] faculty. The CSU will state explicitly what a graduate of the California State University is expected to know” This statement calls for generation of target outcomes and the accompanying assessment tools to gauge educational success at attaining these outcomes. This project has not attempted to address either assessment or specific educational goals, since these components are ancillary to target outcomes.

III. Context

Desired learning outcomes must mirror a comprehensive learning objective. Consequently, the following was first adopted as the baccalaureate objective in mathematical reasoning:

To instill in graduates the ability and desire to use, understand and appreciate mathematics throughout their lives.

The outcomes, conceived to constitute this objective, are phrased so as to be applicable to any CSU baccalaureate graduate, regardless of academic background. Each outcome generally has two aspects; one specific to the needs of each graduate's discipline and the other addressing societal competency common to all graduates. These two, sometimes very different, aspects must be considered when educational goals are drafted in response to the desired outcomes, and again when assessment instruments are developed to measure success at meeting these learning outcomes. Another consideration is that general education and major courses will likely share, though not necessarily equally, in providing both aspects of an individual's mathematical education. A student's major curriculum is not singly responsible for learning related to the discipline, nor is general education singly responsible for societal needs. In other words, general education can enhance the major, but the major can also enhance general education.

According to *Baccalaureate Education in the California State University* (Ref. 3), the "three broad areas of educational achievement expected of CSU graduating students are: (1) acquiring a sophisticated knowledge base, (2) acquiring the skills needed to use knowledge and to learn new knowledge so as to renew their knowledge base, and (3) participating in a mix of collegiate experiences and social processes that contribute to values for successful living." In the case of mathematical reasoning, every CSU student must first develop an extensive palette of mathematical knowledge and skills prior to continuing their education toward the baccalaureate (Refs. 4 and 5). Consequently, baccalaureate education should, as a minimum, foster retention of essential entry-level mathematical knowledge and skills while building upon, enhancing and focusing these base levels within a myriad of applications.

IV. Mathematical Reasoning Learning Outcomes

These outcomes are divided into six categories by topic of achievement: attitude, awareness, ability, understanding, application, and tools. Rationale and explanation are presented in the Appendices.

Baccalaureate graduates of the California State University will be able to;

Attitude

1. view mathematics with heightened interest, increased confidence, and less anxiety as a result of their educational experiences.
2. regard mathematics as a way to think, reason and conceptualize, not simply as a set of techniques.

Awareness

3. recognize when they possess the personal ability to use mathematics within a specific

context.

4. recognize when a specific problem or process may be addressed mathematically, even when they do not personally possess the requisite mathematical ability.
5. seek assistance with and research mathematical concepts, when needed.
6. understand and appreciate the connections between mathematics and a variety of quantitative and non-quantitative disciplines.

Ability

7. understand and perform requisite mathematical calculations and symbolic manipulations.
8. translate verbal statements to and from mathematical expressions.
9. use principles of estimation.
10. use proportions in a variety of contexts, including scaling, rates, unit conversion, and price per unit.
11. develop and apply measurement techniques to data collection, and evaluate potential sources of error, including variability and bias.

Understanding

12. interpret, make appropriate judgments, and draw reasonable conclusions based on numerical, graphical and symbolic information.
13. critically evaluate quantitative information, and identify deceptive or erroneous reasoning.

Application

14. appropriately apply mathematics to represent physical and social processes having personal significance and reflecting a variety of realistic contexts.
15. mathematically evaluate the significance of risk and compare the potential impacts of different hazards.
16. use mathematical-based reasoning to help evaluate alternatives and make decisions.
17. effectively organize, summarize and present information in quantitative forms, including tables, graphs and formulas.

Tools

18. use appropriate technology, including computers and calculators, as tools to assist with numerical and graphical analyses.
19. use technology in an ethical and responsible manner.

V. Conclusions

While the CSU has focused considerable attention on the writing proficiency of baccalaureate graduates, the same cannot be said for mathematics. Currently, the minimum requirement for graduation from the CSU system is the completion of one lower-division course in mathematical reasoning, as decreed by general education breadth requirements (Ref. 6). Simply stated, a single mathematics course will not permit graduates to meet these desired learning outcomes. To do so, *students must be exposed to mathematics at a variety of levels, in many different contexts, and throughout the curriculum.* Furthermore, the mathematical content should be sufficiently challenging to reflect university-level work, and must focus on reasoning founded in the

application of mathematics, not simply based on numbers and arithmetic.

Of course, many students receive higher exposure to mathematics than this minimum, as dictated by their major fields and, to a much lesser extent, by more stringent general education criteria at some campuses. The question remains as to how these outcomes will be achieved for all students. For those majoring in quantitative fields, the desired outcomes may already be satisfied, while for other students this is likely not the case. Even students in quantitative majors may be mathematically deficient, particularly in terms of societal needs, due to specialization in only certain mathematical topics. Mathematical applications in daily life encompass many different fields of mathematics; statistics, algebra, trigonometry, geometry, calculus and finite mathematics are all significant.

Consequently, mathematical literacy must be understood as a campus-wide responsibility, yet must be founded within the major discipline. That is, each major field must define and demonstrate how their students will acquire these competencies through a combination of general education, major discipline, and other courses. These courses do not necessarily have to be math courses, but may be topical courses containing relevant math content.

By necessity, these mathematical outcomes are expressed in general terms which are subject to interpretation. One interpretation might be that these outcomes are no more advanced than expectations for college-bound, high school graduates. For university graduates, minimal expectations should focus on increased mathematical sophistication as each student attains a higher level of cognizance and intellect, the desired product of a comprehensive university education.

It is essential to recognize that these outcomes will not be accomplished in the short term. This will be an evolutionary process, wherein both students and faculty strive for increased proficiency in, and acceptance of, mathematics. Since this concept is in its infancy, further intellectual discussion at a system-wide level is highly desirable. Eventually, individual campuses might choose to adopt similar target outcomes for their graduates. Although to attain these outcomes, adequate and essential resources are needed for both students and faculty. These resources could be modeled on concepts previously developed to enhance writing proficiency. To be considered are concepts such as math across the disciplines, math proficiency courses, math proficiency centers, and others.

This study has focused on only one aspect of student-centered learning, learning outcomes. These outcomes will be meaningless unless subsequent efforts are undertaken to perform assessment and to develop educational goals in response to the assessment results. Learning outcomes are the foundation of a university, but assessment and educational goals are the walls and roof. The entire assembly must be constructed before the structure is complete.

VI. References

1. *CSU Learning Outcomes Proposal*, California State University, Chico, 1998.
2. *The Cornerstone Report*, the California State University, 1996 (available at

www.calstate.edu/cornerstones).

3. *Baccalaureate Education in the California State University*, Academic Senate of the California State University, November 1997 (available at www.calstate.edu/acsenate).
4. *Statement on Competencies in Mathematics Expected of Entering College Students*, June 1997 (available at www.certicc.org).
5. *NAEP 1996 Mathematics Report Card for the Nation and the States*, U.S. Department of Education Office of Educational Research and Improvement, February 1997 (available at www.ed.gov/NCES/naep).
6. *Title 5, California Code of Regulations, Article 5 General Requirements for Graduation, Section 40405 General Education Breadth Requirements*, Board of Trustees of the California State Universities.

Appendix A: Explanation of Outcomes

Baccalaureate graduates of the California State University will be able to;

1. view mathematics with heightened interest, increased confidence, and less anxiety as a result of their educational experiences.

Everyone will encounter times when it will be necessary to call upon their mathematical knowledge to address problems encountered throughout life. If one views math in a positive way, these times will not be viewed as dreaded experiences, but ones where a feeling of growth and accomplishment may be achieved. Similarly, if math is viewed in a positive way, then activities and experiences that involve math will not be avoided solely because they involve some degree of mathematical understanding. This outcome distinguishes a general CSU graduate from the general high school graduate. While the minimal mathematical knowledge requirements are primarily first learned prior to entry, the baccalaureate curriculum should permit sufficient practice in applying this knowledge so as to achieve this outcome.

2. regard mathematics as a way to think, reason and conceptualize, not simply as a set of techniques.

Mathematics should not be learned as a collection of techniques unrelated to general processes of thought, reasoning, and conceptualization. Rather, the logic of mathematics can be used as a model to approach a variety of problems.

3. recognize when they possess the personal ability to use mathematics within a specific context.

Mathematical problems have a broad range of difficulty, and graduates' skills also cover a broad range. Graduates should be able to recognize situations in which they possess the required ability to solve problems, and then be able to use their abilities. This is important, as an individual may not even undertake problems involving mathematical reasoning because of uncertainty about the procedures, even though they may possess the ability to solve it, provided an attempt is made.

4. recognize when a specific problem or process may be addressed mathematically, even when they do not personally possess the requisite mathematical ability.

Not all graduates will have the mathematical ability to solve all problems encountered, but they should recognize when mathematical techniques would be beneficial, so that they may choose to seek appropriate assistance.

5. seek assistance with and research mathematical concepts, when needed.

Graduates should be familiar with resources that may be consulted when mathematical questions arise.

6. understand and appreciate the connections between mathematics and a variety of quantitative and non-quantitative disciplines.

Mathematical ideas are involved in many disciplines. Graduates should be familiar with the connections between aspects of mathematics and other areas, even when the connections may not be immediately evident; for example, geometry and art, logic and philosophy, statistics and history. In particular, they should see connections between mathematics and their major.

7. understand and perform requisite mathematical calculations and symbolic manipulations.

It is imperative that graduates be able to perform basic mathematical operations that are encountered in their day to day activities. They should have the ability to calculate with positive and negative integers, decimals, fractions, and percentages. Furthermore, they should be comfortable with techniques that utilize symbols to represent processes, such as block diagrams and procedural flowcharts. These are the mathematical survival skills necessary for everyday life.

8. translate verbal statements to and from mathematical expressions.

Most problems encountered are not specifically stated in mathematical terms, but when the appropriate mathematical expression or equation can be formulated the problem may have a routine solution. It is also necessary to take problems that are written as mathematical expressions and be able to view these expressions as having some practical meaning, and not just as an abstract assortment of symbols.

9. use principles of estimation.

Estimation involves selecting an appropriate method of approximating a solution and calculating it based on the chosen method. This talent is needed on an almost daily basis. Approximate methods are also valuable when inadequate information is available to fully define a problem.

10. use proportions in a variety of contexts, including scaling, rates, unit conversion, and price per unit.

Problems involving scaling, rates, unit conversions, price per unit, and other applications of proportions are commonly encountered in daily life.

11. develop and apply measurement techniques to data collection, and evaluate potential sources of error, including variability and bias.

Results of data are often given using descriptors of central tendency, such as mean, median and mode, and variability, such as range, variance and standard deviation. Graduates should understand what meaning these numbers convey and their application,

as well as be wary of how these numbers can be used improperly, and even be used to justify a particular view, whether valid or not.

12. interpret, make appropriate judgments, and draw reasonable conclusions based on numerical, graphical and symbolic information.

Information is often given in a variety of mathematical forms. Graduates should be able to gain information from these forms and should also be able to interpret results from polls and surveys.

13. critically evaluate quantitative information, and identify deceptive or erroneous reasoning.

Graduates should not be deceived by the faulty reasoning of individuals trying to justify a particular agenda.

14. appropriately apply mathematics to represent physical and social processes having personal significance and reflecting a variety of realistic contexts.

Mathematics will have applications to graduates' lives in a professional or vocational setting, as well as in everyday contexts and recreational settings. Graduates should be able to apply mathematics to all of these settings.

15. mathematically evaluate the significance of risk and compare the potential impacts of different hazards.

Everyone is faced with choices throughout their lives that have inherent risks involved. Graduates should be able to make informed choices based on sound and rational consideration of potential consequences.

16. use mathematical-based reasoning to help evaluate alternatives and make decisions.

Mathematics can be used to evaluate certain alternatives involved with financial, personal, or professional options. Graduates should have the ability to analyze these choices in order to make appropriate decisions.

17. effectively organize, summarize and present information in quantitative forms, including tables, graphs and formulas.

In order to succeed in our information-based society, graduates must have a solid understanding of using quantitative forms to share information.

18. use appropriate technology, including computers and calculators, as tools to assist with numerical and graphical analyses.

Graduates should have the ability to recognize when appropriate technologies will aid in

computing, evaluating information, or preparing appropriate forms of information, and they should also be able to use the appropriate devices.

19. use technology in an ethical and responsible manner.

As technology becomes more advanced, tools may be easily misused. Tools should not be relied upon to unjustifiably reflect a greater degree of mathematical sophistication than possessed by the user of the tool.

Appendix B: Rationale and Justification

A central responsibility of a university system is to provide all of its students with the capacity to function in society. Because of the enormous impacts of scientific and technological progress, members of today's society are inundated with information presented in a variety of mathematical forms. Virtually every edition of a newspaper and every news broadcast present quantitative information of some sort; including the current unemployment rate, the percent increase in housing prices, the rate of inflation, the results of correlation studies among behaviors, demographics and disease, the median income for specified populations, stock market indexes, and the proportion of the federal budget allocated to education which is allocated to K-12 education. Information is presented to people in many forms, including a variety of tables, graphs, and summary statistics. Each of these types of quantitative information and methods of displaying information can be easily misinterpreted, and can even be presented in such a way as to intentionally promote misunderstanding.

For many mathematical tasks, it is often thought that it is best to rely on an agent, such as a financial advisor or a computer. However, simply turning a task over to an agent raises as many mathematically oriented questions as it eliminates. For instance, under what circumstances is the cost of a financial advisor not worth the expenditure? Trust in financial advisors depends on a person's attitudes toward risk, which involves a good working knowledge of probability and statistical inference. How does one ensure that a financial advisor or computer program is not making errors? If a complex computer program does not handle a special circumstance, does the user know enough about what the program can do and the results that it can generate to adapt and extrapolate to the special circumstance? Can users understand the nature of the calculations, and what the results of those computations mean? As should be clear, delegating the responsibility of performing a mathematical task to others does not eliminate the personal need to reason mathematically in the role of a good citizen. In fact, doing so in a responsible fashion often demands more, rather than less, mathematical sophistication.

Mathematically-based reasoning is not only central for consumers of mathematical information, it is also central in everyday circumstances; such as making a personal or household budget, balancing a checkbook, determining whether it is advantageous to refinance a mortgage, understanding the nature of and being able to calculate compound interest, and determining how many pieces of wood of varying lengths and how many cuts of each piece of wood would result in the most economical buying plan for building a deck. The ability to reason well in mathematical terms is a central property of being a responsible citizen in modern society. Thus, as part of its charge to produce responsible citizens, the CSU has the responsibility for providing all of its students with the mathematical reasoning skills so necessary.

Based on the premise that mathematics is central to advanced education, there are several specific factors which justify development of target outcomes in mathematics for the baccalaureate degree:

- In an attempt to raise standards it is first necessary to define specific target outcomes.
- Stated outcomes give more opportunity for objective and uniform evaluation of students.
- Assessment has become increasingly important, and in order to perform assessment,

well-defined outcomes are needed.

- Stated outcomes give the community an opportunity to examine, criticize and endorse standards.