Math 110 Exam 2
Spring 1999

1. Solve the following boundary value problem: $u_{xy} = 2xy$ for $x > 0$, $y > 0$ with $u(0, y) = \sin y$ and $u(x,0) = e^{3x}$

2. Separate variables by assuming $U(x,t) = X(x)T(t)$ in the boundary value problem below and derive the associated ordinary differential equations in X and T together with their appropriate **boundary** conditions. DO NOT SOLVE THE SEPARATED EQUATIONS

$$U_t(x,t) = U_{xx}(x,t) + 4U_x(x,t), \qquad t>0, \qquad 0 < x < c$$

$$U_x(0,t) = 0 \qquad U_x(c,t) = 0, \qquad 0 < t$$

$$U(x,0) = f(x)$$

3. Find the **<u>bounded</u>** eigenfunctions and associated eigenvalues for the following Sturm-Liouville equations: Note that part B may require the computer.

A.
$$X''(x)+\lambda X(x)=0 \qquad X(0)=0 \qquad X'(\pi)=0, \qquad 0 < x < \pi$$

B.
$$x^2X'' + 2xX' + \lambda x^2 X = 0, \qquad X(\pi) = 0, \qquad 0 < x < \pi$$

4. (a) Use the computer to find the Cosine series for $f(x) = x^2$ on the interval $0 < x < \pi$

(b) What will the series converge to at $x = \pi$?

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5. A. Verify that $u(x,t) = \sum_{n=1}^{\infty} n^3 e^{-n^2 t} \sin(nx)$ converges (uniformly) for all x and all $t > t_0$ where $t_0 > 0$.

B. Verify that $e^{-nkt} \sin nx$ satisfies the boundary value problem:

$$Y_{tt}(x,t) = -k^2 Y_{xx}(x,t) \qquad Y(0,t) = 0, \quad Y(\pi,t) = 0, \quad Y_t(x,0) = -nk\sin nx$$

6. On the interval, $0 < x < \pi$, assume that the following holds:

$$\left(X, L[Y]\right) = \left(L[X], Y\right) \text{ AND } L[X] = 3X \text{ AND } L[Y] = 5Y,$$

PROVE THAT $\left(X, Y\right) = 0$