Math 135 Exam 1 Fall 2001

1. Let

\[ A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 5 & 0 \\ 0 & 3 & -1 \end{pmatrix} \]

a. Find the LU factorization of \( A \)
b. Find a basis for the column space of \( A \).
c. Find the rank of \( A \),

2. Given the following matrices:

\[ E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & -2 & 3 & 8 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

Find bases for the column space, row space, null space, and left null space of matrix \( A \).

3. Suppose matrix \( A \) and its reduced echelon form \( R \) are given as:

\[ A = \begin{pmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

a. What can you say immediately about row 3 of \( A \)?
b. What are the numbers \( a \) and \( b \)?
c. What is a basis for the null space of \( A \)?
d. Circle the spaces that are the same in \( A \) and \( R \):
   - column space, row space, null space, left null space

4. Suppose \( A \) is an \( m \) by \( n \) matrix of rank \( r \).

a. If \( A\vec{x} = \vec{b} \) has a solution for every right side, \( \vec{b} \), what is the column space of \( A \)?
b. In part (a), what are all equations or inequalities that must hold between numbers \( m \), \( n \), and \( r \).
c. Give a specific example of a 3 by 2 matrix of rank 1 with first row \([2, 5] \).
d. Describe the column space and null space of your matrix in part (c).
e. Suppose the right side, \( \vec{b} \), is the same as the first column in your example (part c). Find the complete solution to \( A\vec{x} = \vec{b} \)