1. Find the Jacobian of the transformation.

\[ x = 4a^2 + 10v^2, \quad y = 3u^2 - 9v^2 \]

2. Determine whether or not \( \mathbf{F} \) is a conservative vector field.

Match each vector field in the left column with the appropriate description in the right column.

\[ \mathbf{F} = xe^y \mathbf{i} + ye^x \mathbf{j} \]

conservative

\[ \mathbf{F} = e^x \mathbf{i} + xe^y \mathbf{j} \]

not conservative

3. Use plots of each vector field to determine which is conservative. Then determine whether your guess is correct.

Select the correct answer.

a. \( \mathbf{F}(x, y) = (2xy + \sin y)\mathbf{i} + (x^2 + x\cos y)\mathbf{j} \)

b. \( \mathbf{F}(x, y) = \left( \frac{x - 5y}{\sqrt{x^2 + y^2}} \right) \mathbf{j} \)

4. Evaluate the line integral around the closed curve \( C \) by using Green’s Theorem.

\[ \int_C x^2 \, dx + x^3 \, dy \]

5. Use the Divergence Theorem to calculate the surface integral

\[ \iint_S \mathbf{F} \, dS \]

that is, calculate the flux of \( \mathbf{F} \) across \( S \).

\[ \mathbf{F}(x, y, z) = e^x \sin y \mathbf{i} + e^x \cos y \mathbf{j} + y \mathbf{z}^2 \mathbf{k} \]

\( S \) is the surface of the box bounded by the planes \( x = 0, x = 1, y = 0, y = 2, z = 0, z = 1. \)

Select the correct answer.

a. 4  b. 1  c. 2

6. Use the Divergence Theorem to calculate the surface integral

\[ \iint_S \mathbf{F} \, dS \]

that is, calculate the flux of \( \mathbf{F} \) across \( S \).

\[ \mathbf{F}(x, y, z) = x y \sin z \mathbf{i} + 8 \cos (x z) \mathbf{j} + y \cos z \mathbf{k} \]

\( S \) is the ellipsoid \( \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{16} = 16 \)

7. Find the derivative of the vector function \( \mathbf{r}(t) = e^{t^2} \mathbf{i} - t + \ln (1 + 6t) \mathbf{k} \)

8. Find the length of the curve:

\[ \mathbf{r}(t) = 9t^2 \mathbf{i} + 18t \mathbf{j} + 9 \ln (t) \mathbf{k}, \quad 1 \leq t \leq e \]

Please give the answer to four decimal places.

9. Find the unit tangent vector \( \mathbf{T}(t) \).

\[ \mathbf{r}(t) = < 8t^2, 16t, 8 \ln t > \]
10. A projectile is fired with an initial speed of 843 m/s and angle of elevation \(33^\circ\). Find the range of the projectile.

Select the correct answer.

a. \(d \approx 33\) km  
   b. \(d \approx 27\) km  
   c. \(d \approx 21\) km  
   d. \(d \approx 55\) km

11. The parametric equations \(x = a \sin u \cos v, y = b \sin u \sin v, z = c \cos u, 0 \leq u \leq \pi, 0 \leq v \leq 2\pi\) represent an ellipsoid. Along which axes is the ellipsoid’s semimajor (longest) axis when \(a = 7, b = 3, c = 2\)?

   a. \(x\)-axes  
   b. \(z\)-axes  
   c. \(y\)-axes

12. If \(f(x, y) = 7 - 6x^2 - 8y^2\) choose the correct statement.

   a. \(f_x(6, 3) = -72, f_y(6, 3) = -41\)  
   b. \(f_x(6, 3) = -72, f_y(6, 3) = -48\)  
   c. \(f_x(6, 3) = -41, f_y(6, 3) = -65\)  
   d. \(f_x(6, 3) = -48, f_y(6, 3) = -72\)  
   e. \(f_x(6, 3) = -65, f_y(6, 3) = -41\)

13. Find \(f_y\) for \(f(x, y) = 2x^5 + 9x^3 y^2 + 4xy^4\).

14. Use the linearization \(L(x, y)\) of the function

   \[f(x, y, z) = \sqrt{x^2 + y^2 + z^2}\]

   at \((5, 4, 2)\) to approximate \(f(5.16, 3.95, 2.03)\).

15. Find the direction in which the function \(f(x, y) = x^4 - x^2 y^3\) decreases fastest at the point \((1, 2)\).

16. Evaluate the integral by changing to polar coordinates.

   \[\int \int_R \sqrt{\frac{x^2}{2} + \frac{y^2}{2}} \, dA\]

   where \(R = \{ (x, y) \mid 16 \leq x^2 + y^2 \leq 49, y \geq 0 \}\).

   If entering a decimal, round to the nearest hundredth.

17. Evaluate \(\int_C x^4 \, dx\), where \(C\) is the right half of the circle \(x^2 + y^2 = 9\).

18. Find the exact value of \(\int_C \mathbf{F} \cdot \, dr\), where \(\mathbf{F}(x, y) = e^x - 1 \mathbf{i} + xy \mathbf{j}\) and \(C\) is given by \(\mathbf{r}(t) = t^2 \mathbf{i} + t^3 \mathbf{j}\), \(0 \leq t \leq 3\).

19. Let \(\mathbf{F} = \nabla f\), where \(f(x, y) = \sin(x - 8y)\). Match the curves in the right column with the equation it satisfies in the left column.

   \[\int_C \mathbf{F} \cdot \, dr = 1\]

   \(C\) is the line segment from \((0, 0)\) to \((\pi/2, 0)\)

   \[\int_C \mathbf{F} \cdot \, dr = 0\]

   \(C\) is the line segment from \((0, 0)\) to \((0, \pi)\)

20. The vector field \(\mathbf{F}\) is shown in the \(xy\)-planes and looks the same in all other horizontal planes. (In other words, \(\mathbf{F}\) is independent of \(z\) and its \(z\)-component is 0.) Is \(\text{div} \, \mathbf{F}\) positive, negative, or zero in the following cases?
2. 

3. 

1  
3  
2  

zero  
positive  
negative
1. \( 264u \cdot v \)
2. \( F = \frac{e^y i + xe^y j}{1+6i} \rightarrow \text{conservative} \)
3. a
4. 36
5. c
6. 0
7. \( e^{i \theta} + \frac{6}{1+6i} \cdot k \)
8. \( \begin{pmatrix} 2t^2 \\ 2t + 1 \\ 2t^2 + 1 \\ 1 + 2t + 1 \end{pmatrix} \)
9. \( \begin{pmatrix} 2t^2 \\ 2t + 1 \\ 2t^2 + 1 \\ 1 + 2t + 1 \end{pmatrix} \)
10. a
11. a
12. b
13. \( 10x^4 + 27x^2 - 2y^2 + 4y \)
14. 6.806591
15. \( \frac{8}{11} \)
16. \( \frac{279\pi}{3} \)
17. 291.6
18. \( e^x = \frac{19683}{8} \cdot \frac{1}{e} \)
19. \( \int_C F \cdot dr = 0 \rightarrow C \) is the line segment from (0, 0) to (0, \( \pi \)).
20. 1 \rightarrow \text{negative},
    2 \rightarrow \text{positive},
    3 \rightarrow \text{zero}