I’ll start by drawing the Edgeworth box for two traders (A and B) by superimposing two sets of indifference curves, one of which is inverted via a 180° rotation. This is easily done in the classroom with 2 transparencies.

There are two traders—A and B. And there are two goods—X and Y. The total quantities of X and Y in our little exchange economy are 20 and 10 units, respectively. The traders have conventional preferences. Trader A’s are shown here:

![Indifference curves for Trader A](image)

Trader B has similar preferences, but I am going to rotate his preference map by 180° and I am going to mess with the axis system in the following way. Point Z is at (8, 4). This is going to mean that trader A has eight units of X and 4 units of Y. This is shown with green. But since there are 20 total units of X, trader B must have 12. Likewise, trader B has 6 units of good Y. This is shown in blue. Trader B’s utility goes uphill in the direction of the brown arrow. Note that point Z is now in effect a point in 4-dimensional space.

![Indifference curves for Trader B](image)

On the next page I superimpose these two figures.
This is called the Edgeworth-Bowley box. If trading starts at a point like Z, one of the traders might say, “I’ll give you some of this if you give me some of that,” and so forth. Trading means that you move from point to point in the box. And since the dimensions of the box are fixed, that means that goods are not magically appearing or disappearing. They are just changing hands.

Here is a big question: Is it possible for a trade to happen at Z that will make both traders better off? Think about it before you turn the page.
Sure, any trade that move us to some point in the pink hash-marked area will put both traders at points they like better than Z. They might, for example, trade to point W, with A giving up good Y to get good X, and B giving up good X to take more Y. Now once they are at point W, could further utility increasing trading take place? Yes. They could trade into the green hashed region, which will always be a subset of the pink one. Where will all this lead, where will they be when no further mutually utility improving trading is possible? Think, then turn the page.
They will be somewhere along the red contract curve marked CC. The CC is the set of all points where an A indifference curve is tangent to a B indifference curve, as drawn at point F. So they can trade into better set, which may leave a smaller better set,...until trade leaves them at tangency of two indifference curves--this is a Pareto efficient allocation. Trade does not lead to a unique Pareto efficient allocation--just to a point on the contract curve, unless one of the traders gets stubborn.

Contract curve is the set of all Pareto efficient points

Trading processes

We’ve so far been talking about a barter process. “I’ll give you this, if you give me that.” But
we can also discuss this in a situation where there are prices and budget constraints.

Suppose there is an “auctioneer” who announces that the prices for good X and Y are both $1. This keeps the picture simple.

Suppose the market opens with the initial endowment point where A has (5, 6) and B has (15, 4). This is point Z above. It is not hard to see that the red line is A’s BC. It is a little harder, but not too hard to see that this red line is also B’s budget constraint. One line...two budget constraints. Pretty cool. So here’s another question. If we start at Z and the auctioneer calls out the prices Px = 1 and Py = 1, are we in an equilibrium? Does supply equal demand? Try to figure that by looking at the above, and then turn the page.
Those are not equilibrium prices. A will have his optimum at a point like A0 and B will have her optimum at B0. A’s demand for good X is less than B’s supply for good X, and not coincidentally, B’s demand for good Y exceeds A’s supply. This is due to Walras’ Law, as you will see.

So what will the auctioneer do? Reduce Px and increase Py. If we are still at Z, and no trading has yet taken place, this will rotate the budget constraint through point Z in a counter-clockwise direction.

So what would an equilibrium look like? That is, if prices move until an equilibrium is found, what would the picture above look like? Think, then turn.
It happens when both traders want to go to the same point on the BC, as you see above. And that’s a point on CC, on the contract curve, and it is a Pareto optimum. We have a very big proof here: A COMPETITIVE EQUILIBRIUM IS A PARETO OPTIMUM.

**First theorem of welfare economics**: all market equilibria are Pareto efficient. (Assumes No externalities, etc. etc.) Proof: The indifference curves are tangent at the equilibrium and at the Pareto optimum.

Will such a point always exist if we have well behaved preferences? Think, then turn.
Consider the figure here. Every point on the CC defines a line which is tangent to the two indifference curves, which are tangent to each other on CC. The two dashed green lines and the solid brown line are such lines. Let W be the endowment point. The tangent line at X falls below W, due to the convex preferences of trader A. The tangent line at Y runs above W because of the convex preferences of trader B. The tangent lines change continuously, by continuity of preferences of A and B. Checking the tangent lines as we move along the contract curve from X to Y, we go from below W to above W, hence there must be a line that goes through W and is tangent to both indifference curves. This proves the existence of an equilibrium, which is also, by the way, Pareto optimal.

UNIQUENESS ...I’ll discuss

STABILITY ...I’ll discuss.

Second welfare theorem: if all agents have smooth convex preferences, then there will always be a set of prices such that each Pareto efficient allocation is a market equilibrium for an appropriate assignment of endowments.
Budget Constraints and Walras’ Law

The central idea with respect to budget constraints is that

\[ \text{sources of purchasing power} = \text{uses of purchasing power} \]

or

\[ \text{the value of what the trader enters the market with} = \text{value of what trader leaves market with} \]

The first budget constraint you normally see (maybe in econ 101) is something like

\[ \text{income} = P_x X + P_y Y, \]

where \(X\) and \(Y\) are the quantities bought of two goods, \(P_i\) is the price of good \(i\), and the trader must spend all of his income on either good \(x\) or good \(y\).

Suppose that trader \(j\) enters the market with the quantity \(x'ji\) of good \(i\), and he leaves the market with the quantity \(xji\). If there are \(n\) goods plus money, then trader \(1\)'s budget constraint will look like this:

\[ P_1 \cdot x'11 + P_2 \cdot x'12 + \ldots + P_n \cdot x'1n + M'1 = P_1 \cdot x11 + P_2 \cdot x12 + \ldots + P_n \cdot x1n + M1 \]

where \(M'1\) is trader \(1\)'s initial holding of money, and \(M1\) is his desired ending holding.

Think about the meaning for the term \(x12 - x'12\). This is the quantity of good 2 that trader 1 wants, minus the quantity of good 2 that trader 1 has. In other words, it is trader 1's excess demand for good 2. (Since we are contemplating a trader who faces only a budget constraint and no other limitations (like surpluses or shortages) on his trading, the excess demand should be called a “notional” excess demand. Let us write \(x12 - x'12\) as \(Z12\). This definition lets us rewrite the above equation more compactly as:

\[ 0 = P_1 \cdot Z11 + P_2 \cdot Z12 + \ldots + P_n \cdot Z1n + M1 - M'1 \]

Assume that there are \(t\) traders, each one having a budget constraint, then all of these budget constraints would look like this:

\[ 0 = P_1 \cdot Z11 + P_2 \cdot Z12 + \ldots + P_n \cdot Z1n + M1 - M'1 \]

\[ 0 = P_1 \cdot Z21 + P_2 \cdot Z22 + \ldots + P_n \cdot Z2n + M2 - M'2 \]

\[ \ldots \]

\[ 0 = P_1 \cdot Zt1 + P_2 \cdot Zt2 + \ldots + P_n \cdot Ztn + Mt - M't \]
Now

\[ \text{Define } Z_i = \sum_{j=1}^{i} Z_{ji} \]

Thus, \( Z_i \) is the *market* excess demand for good \( i \).

If we add all the budget constraints and group terms appropriately, we get

\[ 0 = P_1 \cdot Z_1 + P_2 \cdot Z_2 + \ldots + P_n \cdot Z_n + M - M' \]

This is Walras’ Law. It says that the sum of the values of all notional market excess demands (including money) is exactly zero. This implies some interesting things:

1. If there is a negative notional market excess demand (i.e., and “excess supply”) for some item, then there must be a positive notional market excess demand for some other item.

2. A generalized, system-wide “excess supply of everything” is impossible.

3. If one could show that notional market excess demands are zero for all goods but one, it would be possible to conclude that the remaining good also has a zero notional market excess demand. In this case the market would be in general equilibrium. This can be a labor saving device for a theorist.

4. If \( M < M' \), i.e., money demand is less than money supply, then there is a net positive notional excess demand for other items.

What can we say about the sum of the values of all market effective demands? Nothing...more on this later.
We are at a critical point here. Since the time of Adam Smith, people have thought that something very good happens with competitive markets...they go to equilibrium, and the equilibrium is a good place. Smith didn’t know about contract curves, Pareto optima, etc, he was reduced to waving his arms and babbling about “invisible hands”...but he was very right about the virtues of competitive markets (in certain cases) & years of theory have sharpened our focus.

But sometimes free markets turn into monopolies....what does our technique have to say about that?

**monopoly**

Suppose one of the traders, A, gets to dictate the prices. He will pick a point on B’s offer curve tangent to A’s indifference curves. See fig 31.5. Note the solution is not Pareto optimal. See also the perfect price discriminating monopolist in fig 31.6, which is Pareto optimal. The book will get you through this stuff just fine.
Suppose traders’ endowments put them at point W, which is off the contract curve, the dashed black line marked C. And the current prices establish the red budget constraint. Trader A would maximize his utility by trading to point M and supplying good X in the quantity indicated by the green arrow. You might have to stand on your head to see it, but trader B will want to go to point V and buy good X in the quantity shown by the mustard colored arrow. It is important that you understand why V is to the left of T (when you are not standing on your head). Note that convexity of the U curves forces these results. There is, therefore, an excess demand for good X, so the “auctioneer” will increase Px. Think for a second...what does Walras’ Law imply about good Y. Ans: It is in excess supply, so the auctioneer will cut its price.

If no trading has yet taken place, these price adjustments will cause the BC to rotate clockwise through point W. You could construct a similar argument by drawing a BC through W and Z...you would find that the excess demands then have the opposite signs and that the BC would rotate counterclockwise through W from that position.

Since all the functions used are continuous, you know that if, as you rotate the BC clockwise through W beginning with the red one, good X goes from a condition of excess demand to excess supply...it therefore passes through zero, which means the good X market clears or is in equilibrium. Then Walras’ Law implies that the good Y market is also in equilibrium...which means we are at a point like F, on the contract curve, at a Pareto Optimum, and at a competitive equilibrium. This is how the auctioneer/market supposedly brings about good things.
Trader A, Axel, lives on a desert island. Every week a CARE package floats ashore containing 10 pounds of yak meat and 10 pounds of oreos. He is indifferent to the oreos. Every week Axel tosses his oreos, so to speak, and eats all the yak. His indifference curves look like this, where W is his endowment point.

One day a guy named Bob (Trader B) washes ashore. Bob likes both oreos and yak. Amazingly, two care packages float ashore each week after Bob arrives. Axel grabs one and Bob grabs the other. At first they engage in mutually advantageous trading, which leads them to some point on the line segment HK, where Axel trades away all his oreos. (Can you see why? Look at the intersection of their better sets.) But over time Axel grows to dislike Bob. He especially does not like to see Bob enjoying his oreos. This effects Axel so much that it bends his indifference curves. He now attaches some value to keeping oreos, because now any oreo that Axel keeps is one that Bob won’t get, and Axel likes that. As a result, oreos become a “good” to Axel, and his indifference curves take on the usual “well behaved” look.
Mutually advantageous trading will now take them to somewhere, like point X on the contract curve CC. This is “Pareto Optimal”...BUT if Axel weren’t such a jerk, they would both be better off on the segment LM. Alex needs therapy. His preferences aren’t “right.” What does this say about utility analysis?

Lesson1: Pareto optimality might not be a very pretty place.
Lesson2: It doesn’t pay to live with an asshole.
Lesson 3: It might not even pay to *be* an asshole.
Here’s another case: I currently have Ma dollars, and I am thinking about painting my house. Right now it is sort of a violet color. I am on the second U curve at point X. (The horizontal axis is the wavelength of visible light, i.e., the color.) If I pay to paint my house green, my money level will fall to M’a, but my U level will rise, because I like that color more than the money. I would be at point Y, and Ua(Y) > Ua(X).

You live around the corner from me. You watch the painters get ready to paint. You care what color my house is. You want it to be somewhere between yellow and red. You also have some money. We can make an Edgeworth/Bowley box with our combined money (after I pay for the painting) on the vertical axis and the color of my house on the horizontal.

If I were to paint it my favorite color, we are not at a Pareto optimum. The Pareto optimum points are on line Cc. We would both be better off if you would pay me to adjust my color toward red. I’d like it better with more violet, but if you are willing to pay enough, I’ll change the color. That would be Pareto improving.

If you take the Pareto optimum to be a sort of ethical imperative, then I “should not” paint my house the color I want. It is true that we would both be better off if you bribed me to paint it another color. But maybe you ought to just mind your own business. Maybe you ought to just flatten out your indifference curves. Maybe we’d be better off that way. I’d get the color I want, and you’d keep your money.

The real problem surfaces when you think you have a “right” to tell me what color I can paint my house—and I have to pay for the paint job. That would put us at point Z. Again, it would be Pareto improving if I could bribe you to let me paint it the color I want, and move to the right of Z, but... Notice that there is an “externality” here, but there is noting in economic analysis that will clarify who has a “right” to what.
The miracle of the market...or how the market can peacefully co-ordinate the actions of 270 million selfish little pigs.

At a Pareto optimum, i.e., on the contract curve, MRSa = MRSb for all pairs of traders a and b.

When prices are posted, each trader *tries* to set his/her MRS = -P1/P2.

At a competitive equilibrium, each trader *can* actually make all desired trades, since there are no shortages or surpluses.

So if we are at an equilibrium, we have MRSa = -P1/P2 and MRSb = -P1/P2. Therefore, MRSa = MRSb, and we are on the contract curve and at a Pareto optimum.

Note the role of prices: By adjusting to them, we adjust to each other. Prices co-ordinate our actions the way a drummer keeps the band (or the dancers) in synch...the way the highly accurate clocks in GPS satellites can co-ordinate the clock in your GPS receiver, sort of.

It is important to see that only *equilibrium* prices do this...disequilibrium prices screw everything up...like a bad drummer.