Math 235 Topics and Problems--Chapters 1-5

Definitions:

- Vector space
- subspace
- linear combination
- linearly dependent
- linearly independent
- span
- basis
- dimension
- row equivalent
- echelon form
- free variable
- row rank
- rank
- homogeneous
- consistent
- \( S + T \)
- \( S \cap T \)
- \( L(V, V) \)
- domain
- nullity
- null space
- invertible
- isomorphism
- inner product
- orthonormal basis
- orthogonal projections
- Gram-Schmidt Orthogonalization
- orthogonal transformation
- determinants
- change of basis matrix

Finding distance point to line, point to plane. Proving an orthogonal set of vectors is linearly independent o other testing for dependence/independence -- related results / theorems

Proving consequences of the definition of determinant. Calculating a determinant using properties. Proving uniqueness—parts of existence. Proving \( \det(AB) = \det(A) \det(B) \)

Proof that n vectors in \( \mathbb{R}^n \) are dependent IFF \( \det = 0 \).

Solving systems of equations and expressing the solution algebraically and geometrically.

Creating systems of equations with given solutions (or basis for the solution space).

Determining the dimension and finding a basis for \( S \cap T \)

Considering \( A\vec{x} = \vec{b} \) or \( T(\vec{x}) = \vec{b} \): When is \( \vec{b} \) in the range of \( T \)? When is the solution unique? e.g. \( T(x, y) = (x + y, x - 2y, 2x - y) \); \( \vec{b} = \begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix} \)

Interpreting RREF: rank, nullity, basis for solution space, describing the solution space, determining invertibility, 1-to-1, onto, consistency.

Given a transformation in some form, find the matrix representation relative to some odd basis.

e.g. if \( T(x, y) = (2x - y, x + y) \). Find the matrix representation of \( T \) relative to a) the standard basis \( \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \) and b) relative to the basis: \( \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix} \).

Know the statements of important theorems:
Know the proofs of certain important results or parts of theorems:

Suppose \( \{ \vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n \} \) are independent. Explain.

If \( \{ \vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n \} \) is a basis for \( \mathbb{R}^n \) prove that if \( \{ \vec{w}_1, \vec{w}_2, \ldots, \vec{w}_n \} \) is a basis for \( \mathbb{R}^n \) IFF \( A \) is invertible.

Suppose that \( T: \mathbb{R}^m \rightarrow \mathbb{R}^n \) is a linear transformation. Prove that \( \{ \vec{x} \mid T(\vec{x}) = \vec{0} \} \) is a subspace of \( \mathbb{R}^m \).

Suppose that \( A \) is a 4 X 7 matrix. Prove that the columns of \( A \) form a dependent set.

Prove that the range of a linear transformation is always a subspace of the co-domain (range space).

If \( A \) is invertible, prove that the columns of \( A \) are independent.